USE OF A COMPUTER TO INVESTIGATE THE CRYSTALLIZATION OF A REFINED INGOT

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The heat propagation problem is formulated for the crystallization of an ingot obtained by melting metal in water-cooled cylindrical crystallizers.

Modern methods of refining steel (electroslag, electron-beam, vacuum-arc) in water-cooled crystallizers involve the calculation of the temperature fields in the ingot and in the liquid metal bath. We will examine two methods of approximately calculating the temperature fields associated with ingot crystallization: a numerical method and an analytic method. To find a numerical solution we assume that it is possible to take convective heat transfer into account by introducing an effective value of the thermal conductivity in the liquid metal. This makes it possible to reduce the problem of heat and mass transfer to a problem of the Stefan type with essentially discontinuous coefficients.

1. **Formulation of the problem for large crystallizers.** In the region shown in Fig. 1 we seek the solution of the equation

$$\frac{\partial W}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial U}{\partial r}\right) + \frac{\partial}{\partial z}\left(\lambda\frac{\partial U}{\partial z}\right),$$

$$0 < r < R, \quad 0 < z < \eta(t) \tag{1}$$

with the following boundary and initial conditions:

$$\left.\frac{\partial U}{\partial r}\right|_{r=0} = 0, \quad -\lambda\left.\frac{\partial U}{\partial r}\right|_{r=R} = \alpha_1(z)(U - T_1), \tag{2}$$

$$\lambda\left.\frac{\partial U}{\partial z}\right|_{z=0} = \alpha_2(U - T_2),$$

$$\lambda\left.\frac{\partial U}{\partial z}\right|_{z=\eta(t)} = q(r) - \frac{d\eta}{dt}c\gamma(U - T_0), \tag{3}$$

$$U(r, z, 0) = U_0, \quad \eta(0) = \eta_0, \tag{4}$$

where $W = c\gamma U + \rho\gamma e(U - U^*)$,

$$e(U - U^*) = \begin{cases} 1 \text{ when } U - U^* > 0, \\ 0 \text{ when } U - U^* < 0, \end{cases}$$

$$\lambda = \lambda(U) = \begin{cases} \lambda_1(U) \text{ when } U < U^*, \\ \lambda_e \text{ when } U \geq U^*. \end{cases}$$

2. **Numerical method.** To construct the difference schemes of the numerical solution we chiefly employed the results of [1] and [2]. The idea consists in first smoothing the discontinuous functions of W and λ on a certain temperature interval $U^* - \Delta \leq U \leq U^* + \Delta$ with subsequent application of the local one-dimensional method for numerically solving the two-dimensional smoothed problem.

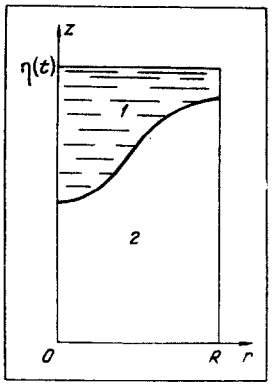

Fig. 1. Region of solution of the problem: 1) liquid phase; 2) solid phase.

We took the following iterationless difference scheme with first order of approximation:

$$\frac{w_{i,j}^{(k+0.5)} - w_{i,j}^{(k)}}{\tau} =$$

$$= \frac{1}{r_i h^2}\left[\lambda^{(k)}_{i+\frac{1}{2},j}\left(r_i + \frac{h}{2}\right)(u_{i+1,j}^{(k+0.5)} - u_{i,j}^{(k+0.5)} - \right.$$

$$\left. - \lambda^{(k)}_{i-\frac{1}{2},j}\left(r_i - \frac{h}{2}\right)(u_{i,j}^{(k+0.5)} - u_{i-1,j}^{(k+0.5)})\right],$$

$$i = 0, 1, \ldots, n, \quad j = 0, 1, \ldots, m_k,$$

$$m_{k+1} = m_k + 1, \tag{I}$$

$$u_{0,j}^{(k+0.5)} = u_{1,j}^{(k+0.5)},$$

$$\lambda^{(k)}_{n-\frac{1}{2},j}\frac{u_{n,j}^{(k+0.5)} - u_{n-1,j}^{(k+0.5)}}{h} = \alpha_{1,j}(u_{n,j}^{(k+0.5)} - T_1),$$

$$\frac{w_{i,j}^{(k+1)} - w_{i,j}^{(k+0.5)}}{\tau} =$$

$$= \frac{1}{l}[\lambda^{(k)}_{i,j+\frac{1}{2}}(u_{i,j+1}^{(k+1)} - u_{i,j}^{(k+1)}) - \lambda^{(k)}_{i,j-\frac{1}{2}}(u_{i,j}^{(k+1)} - u_{i,j-1}^{(k+1)})],$$

$$\lambda^{(k)}_{i,m}\frac{u_{i,m}^{(k+1)} - u_{i,m-1}^{(k+1)}}{l} = q(r_i) - \frac{l}{\tau}c\gamma(u_{i,m}^{(k+1)} - T_0),$$

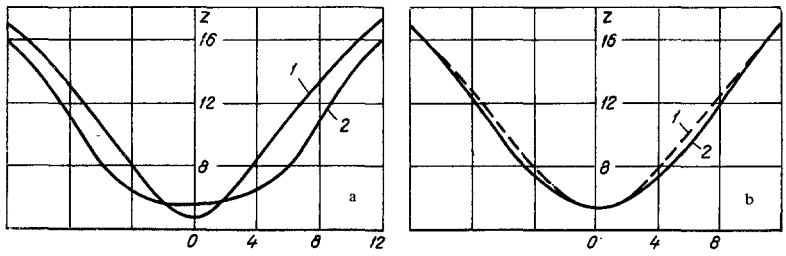

Fig. 2. Effect of the quantity λ_e on the shape of the isothermal surface U* = U for the case of electroslag melting of ShK-15 steel (t = 2400 sec, melting regime: dn/dt = 0.02 cm/sec = const, effective thermal energy introduced 40 kW, flux ANF6. The calculations were made for $\alpha_2 = 2\alpha_1 = 1800$ W/m^2 · deg, U* = 1450° C, $\lambda_s = 29.3$ W/m · deg, $c\gamma = 5$ J/cm^3 · deg) a) curve 1 corresponds to calculations at $\lambda_e = 2100$ W/m · deg, 2 to 210 W/m · deg; b) 1—experimental curve, 2—calculations at $\lambda_e = 417$ W/m · deg.

$$\lambda^{(k)}_{i,\frac{1}{2}} \frac{u^{(k+1)}_{i,1} - u^{(k+1)}_{i,0}}{l} = \alpha_{2,i}(u^{(k+1)}_{i,0} - T_2),$$

$$j = 0,\ 1,\ \ldots,\ m_k,\quad i = 0,\ 1,\ \ldots,\ n, \qquad \text{(II)}$$

where τ, h, and l are the steps with respect to time and the space coordinates r and z, respectively. We ensure that the scheme is iterationless by determining the thermal conductivity only with respect to the temperatures on previous integral time layers and also by calculating the smoothed function $\tilde{e}(U - U^*)$ from the temperatures of the preceding layer. The remaining notation corresponds basically with that employed in [1] and [3].

The calculation scheme of the local one-dimensional method consists in alternately solving sets of one-dimensional problems (I) and (II) and ensures that the rounding errors do not increase during the calculation.

3. Analytic solution. In a series of cases it is desirable to have approximate analytic relations which can be used to estimate the individual characteristics of the thermal field in the ingot as a function of the starting parameters of the process. The solution presented below was obtained with the following additional assumptions for the problem formulated in section 1: 1) λ, c, γ are constants, 2) $d\eta/dt = v = \text{const}$, 3) the presence of a bottom plate can be simulated by continuing the ingot to a great length, 4) there are no latent heat sources, 5) the right side of the second of boundary conditions (3) is represented in the form

$$q - \frac{d\eta}{dt} c\gamma U + q_1(r),$$

where q_1 has a normal radial distribution, i.e.,

$$q_1(r) = \frac{q_0}{\pi R^2 [1 - \exp(-k)]} \exp\left[-k\left(\frac{r}{R}\right)^2\right],$$

$$q = \text{const},$$

k is the concentration coefficient of the heat flux $q_1(r)$. With these assumptions the solution takes the form

$$U(r, z, t) = U_0 + \frac{q}{\pi R \lambda} S_2(r, z, t) + \frac{q_0}{R\lambda} S_1(r, z, t).$$

Here,

$$S_2(r, z, t) = 2\mathrm{Bi} \sum_{n=1}^{\infty} \frac{J_0(\mu_n \bar{r})}{(\mathrm{Bi}^2 + \mu_n^2) J_0(\mu_n)} K_n(\bar{x}, \infty)\, \Psi_n,$$

$$S_1(r, z, t) =$$

$$= \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{J_0(\mu_n \bar{r})\, \mu_n^2}{(\mathrm{Bi}^2 + \mu_n^2) J_0^2(\mu_n)} \exp\left[-\frac{\mu_n^2}{4k}\right] \times$$

$$\times K_n(\bar{x}, \infty)\, \Psi_n(\bar{x}, \mathrm{Fo}),$$

where

$$\bar{r} = \frac{r}{R},\quad \mathrm{Bi} = \frac{\alpha R}{\lambda},\quad \bar{x} = \frac{vt - z}{R},$$

$$\alpha = \alpha_1 = \alpha_2 = \text{const},\quad a = \frac{\lambda}{c\gamma},\quad \mathrm{Fo} = \frac{at}{R^2},$$

$$K_n(\bar{x}, \infty) = \frac{\exp[-\bar{x}(C_n - \mathrm{Pe})]}{C_n + \mathrm{Pe}},$$

$$\Psi_n = \frac{1}{2}\left\{ \operatorname{erfc}\left[\frac{\bar{x}}{2\sqrt{\mathrm{Fo}}} - C_n\sqrt{\mathrm{Fo}}\right] - \frac{\exp(2\bar{x}C_n)}{(C_n - \mathrm{Pe})^2} \times \right.$$

$$\times \operatorname{erfc}\left(\frac{\bar{x}}{2\sqrt{\mathrm{Fo}}} + C_n\sqrt{\mathrm{Fo}}\right) +$$

$$+ 2\mathrm{Pe}\,\frac{C_n + \mathrm{Pe}}{\mu_n^2} \exp[\bar{x} - \mu_n^2 \mathrm{Fo}] \times$$

$$\left. \times \operatorname{erfc}\left(\frac{\bar{x}}{2\sqrt{\mathrm{Fo}}} + \mathrm{Pe}\sqrt{\mathrm{Fo}}\right)\right\},$$

$$\mathrm{Pe} = \frac{vR}{2a},\quad C_n = \sqrt{\mu_n^2 + \mathrm{Pe}^2},$$

μ_n are the roots of the characteristic equation

$$\mathrm{Bi} = \frac{\mu_n J_1(\mu_n)}{J_0(\mu_n)},$$

$J_m(\mu_n)$ is a Bessel function of the first kind of order m with real argument.

To make the above solution convenient for practical use, the functions S_1 and S_2 were tabulated on a computer.

4. Discussion of the results. The numerical method was programmed for a M-20 computer. The program consists of 250 instructions and makes it possible to find the temperature field in a network region of 3000 points. When the number of nodes is varied from 200 to the maximum, the computation time for a single variant is 20 min. For convenience in processing the results the program was supplied with a special subroutine which printed out directly the coordinates of the intersections of the given isothermal lines and the lines of the network.

The results of the numerical experiments showed that convective mixing of the liquid metal is decisive in shaping the heat flow in the metal bath. Characteristic profiles (Fig. 2) of the crystallization surfaces can be obtained when the convective heat transfer is taken into account. Numerical values of the effective thermal conductivity can be estimated starting from the velocity field distribution or by comparing test data with the results of numerical experiments. If some component of the velocity vector predominates over the rest the thermal conductivity may be anisotropic.

Under normal melting conditions an increase in crystallizer radius involves an increase in λ_e. Deviation from normal conditions requires an additional increase in λ_e (intensified heat input) or a reduction in λ_e (cooler regime). Depending on the size of the crystallizer and the melting regime λ_e varies from $10\lambda_s$ to $40\lambda_s$, where λ_s is the mean value of the thermal conductivity in the solid state.

The introduction of an effective thermal conductivity chiefly affects the shape of the metal bath, almost without altering its depth. This makes it possible to use the analytic solution to predict the depth of the metal bath during the melting process.

NOTATION

$W = W(r, z, t)$ is the enthalpy; $U = U(r, z, t)$ is the temperature; U^* is the phase transition temperature; c, γ are the specific heat and specific weight of the material; $\eta = \eta(t)$ is the variable position of the moving outer boundary; ρ is the latent heat; α_1, α_2 are the heat transfer coefficients at the walls and bottom of the crystallizer; $\lambda = \lambda(U)$ is the thermal conductivity; T_1, T_2 are the temperatures of the cooling water; T_0 is the temperature of the metal reaching (uniformly with respect to r) the boundary $z = \eta(t)$; Δ is the smoothing interval; $q = q(r)$ is the heat flux at the boundary.

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